PARAMETRIC METHOD OF SOLVING HEAT-TRANSFER PROBLEMS WITH LIQUID FILM FLOW

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The authors describe a method of calculating the convective heat transfer in a flowing film of viscous liquid.

Most heat-transfer problems with film flow reduce to the study of thin films ($h \ll L$) of a viscous incompressible liquid flowing freely over a heated surface. The liquid motion is considered to be hydrodynamically stable, which a constant film thickness. In spite of the different ways of assigning heat-transfer conditions on the free film surface and the heated surface, the heat transfer in this case is accomplished by conduction [1].

In this paper we consider laminar wave-free motion of a liquid film, accounting for inertia forces and convective heat transfer; we neglect the influence of capillary forces. As boundary conditions for the heat transfer we assume that the temperatures of the film surface and of the solid heated surface are arbitrary functions of the longitudinal coordinate, either given or determined later from the heat-transfer conditions. The motion and convective heat transfer in thin liquid films are usually described in the boundary-layer approximation, i.e., by equations of parabolic type. The difference from the boundary-layer equations is that neither the velocity distribution at the outer edge nor the film thickness is known. The conservation of the mean specific flow rate in the liquid film is an additional relation for calculating the unknown film thickness. The proposed solution of problems of motion and heat transfer in film flow is based on the parametric similarity method, suggested by L. G. Loitsyanskii in boundary-layer theory [2]. The parametric method assumes the introduction of a number of dimensionless groups (parameters) describing the different conditions of motion and heat transfer (e.g., the influence of inertia forces, that the heated surface is not isothermal, etc.). According to the general ideas of the method, the parameters are taken as new independent variables, and the equations of motion and heat transfer in thin liquid films are written in "universal" form. The solutions of these equations, which are independent of the specific features of the problems, can be obtained once and for all and tabulated as functions of the parameters introduced. Depending on the specific conditions of the flow processes the parameters can have different values, but the distributions of velocity and temperature in the "universal" form will be unique for a wide class of film flow problems.

The steady-state equations of motion and convective heat transfer of a thin liquid film, written in terms of a stream function Ψ, have the form:

$$\frac{\partial\Psi}{\partial y}\frac{\partial^2\Psi}{\partial x\partial y}-\frac{\partial\Psi}{\partial x}\frac{\partial^2\Psi}{\partial y^2}=g+\nu\frac{\partial^3\Psi}{\partial y^3}, \tag{1}$$

$$\frac{\partial\Psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}$$

with the boundary conditions

$$\text{for}\quad y=0\quad \Psi=\frac{\partial\Psi}{\partial y}=0,\ T=T_W(x), \tag{2}$$

$$\text{for}\quad y=h(x)\quad \frac{\partial^2\Psi}{\partial y^2}=0,\ \Psi=Q_0,\ T=T_S(x).$$

We introduce the dimensionless variables

$$\eta=y/h(x),\ \Phi=\Psi/Q_0,\quad \Theta=\frac{T-T_S(x)}{T_W(x)-T_S(x)}. \tag{3}$$

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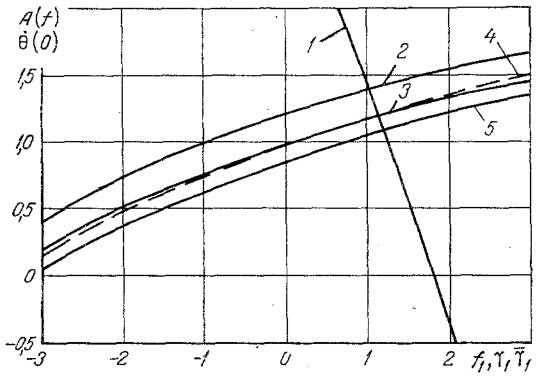

Fig. 1. The universal functions $A(f_1)$ (curve 1) and the specific heat flux to the heated surface (curves 2-5) as a function of f_1, γ_1, $\bar{\gamma}_1$: 2) $f_1 = 0$, $\bar{\gamma}_1 = +1$; 3) $f_1 = -10$, $\bar{\gamma}_1 = 0$; 4) $f_1 = 0$, $\bar{\gamma}_1 = 0$; 5) $f_1 = 0$, $\bar{\gamma}_1 = -1$.

In these variables Eqs. (1) and boundary conditions (2) take the form

$$-\frac{Qh'}{\nu}\left(\frac{\partial\Phi}{\partial\eta}\right)^2+\frac{Qh}{\nu}\left[\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial x\partial\eta}-\frac{\partial\Phi}{\partial x}\frac{\partial^2\Phi}{\partial\eta^2}\right]=\frac{gh^3}{\nu Q}+\frac{\partial^3\Phi}{\partial\eta^3},$$

$$\frac{\partial\Phi}{\partial\eta}\left(\Theta\frac{T_W'}{T_W-T_S}\frac{Qh'}{\nu}+(1-\Theta)\frac{T_S'}{T_W-T_S}\frac{Qh}{\nu}\right)+\frac{Qh}{\nu}\left[\frac{\partial\Phi}{\partial\eta}\frac{\partial\Theta}{\partial x}-\frac{\partial\Phi}{\partial x}\frac{\partial\Theta}{\partial\eta}\right]=\frac{1}{\Pr}\frac{\partial^2\Theta}{\partial\eta^2},$$

$$\eta=0\quad \Phi=\frac{\partial\Phi}{\partial\eta}=0,\ \Theta=1;\ \eta=1\quad \frac{\partial^2\Phi}{\partial\eta^2}=0,\ \Phi=1,\ \Theta=0,$$

where the symbol ' indicates differentiation with respect to x. We introduce the groups $f_1=\frac{Qh'}{\nu}$, $\gamma_1=\frac{T_W'\Pr}{T_W-T_S}\frac{Qh}{\nu}$, $\bar{\gamma}_1=\frac{T_S'\Pr}{T_W-T_S}\frac{Qh}{\nu}$ as a basis for construction of three series of parameters:

$$f_K=\frac{Q^K h^{K-1}}{\nu^K}\frac{d^K h}{dx^K},\ \gamma_K=\frac{Q^K h^K \Pr^K}{\nu^K(T_W-T_S)}\frac{d^K T_W}{dx^K},$$
$$\bar{\gamma}_K=\frac{Q^K h^K \Pr^K}{\nu^K(T_W-T_S)}\frac{d^K T_S}{dx^K}\quad (K=1,\ 2,\ \ldots),\qquad (4)$$

which we take as the new independent variables. In [3] we described the construction of recurrence relations for the parameters and a detailed transition in Eq. (3) from the $\{x;\eta\}$ space to the $\{f_K,\ \gamma_K,\ \bar{\gamma}_K;\ \eta\}$ parameter space. The general form of Eq. (3) with boundary conditions in universal form will be as follows:

$$\dddot{\Phi}=-A-f_1\dot{\Phi}^2-\ddot{\Phi}L(\Phi)+L(\dot{\Phi})\dot{\Phi},$$
$$\ddot{\Theta}=\dot{\Phi}[\Theta\gamma_1+(1-\Theta)\bar{\gamma}_1]+\dot{\Phi}L_1(\Theta)-\dot{\Theta}L_1(\Phi),\qquad (5)$$
$$\eta=0\quad \Phi=\dot{\Phi}=0,\ \Theta=1;\ \eta=1\quad \ddot{\Phi}=0,\ \Phi=1,\ \Theta=0,$$

where $A = gh^3/\nu Q$; $L = [(K-1)f_1f_K + f_{K+1}]\,\partial/\partial f_K$; $L_1 = [(K-1)f_1f_K + f_{K+1}]\Pr\,\partial/\partial f_K + (Kf_1f_K\Pr + \gamma_{K+1} - \gamma_K\gamma_1 + \gamma_K\bar{\gamma}_1)\,\partial/\partial\gamma_K + (Kf_1\bar{\gamma}_K\Pr + \bar{\gamma}_{K+1} - \bar{\gamma}_K\gamma_1)\,\partial/\partial\bar{\gamma}_K$; the subscript K indicates summation; the points denote differentiation with respect to η. The value of $A(f_K)$ is determined from the fourth supplementary condition $\Phi(1) = 1$.

The solution of the universal equations (5) can be obtained only with a finite number of parameters, i.e., in parametric approximations. The simplest for integration are local similarity approximations in which partial derivatives with respect to the parameters are discarded. By considering different groups of parameters, one can analyze the influence of various factors on the nature of the motion and the heat transfer in the liquid films. For instance, the parameters f_K account for the influence of inertia forces and determine the change in film thickness along x due to their influence, and the parameters γ_K and $\bar{\gamma}_K$ account for the influence of temperatures on the outer film boundary and of the solid heated surface on the heat transfer. We call the following equations the one-parameter approximation to the equation of motion and the locally-one-parameter approximation to the heat-transfer equation in the parameters f_1, γ_1, $\bar{\gamma}_1$:

$$\dddot{\Phi} = -A - f_1\dot{\Phi}^2, \quad f_2 = f_3 = \ldots = f_k = 0,$$
$$\ddot{\Theta} = \dot{\Phi}[\Theta\gamma_1 + (1-\Theta)\bar{\gamma}_1], \quad \gamma_2 = \bar{\gamma}_2 = \ldots = \gamma_k = \bar{\gamma}_k = 0. \tag{6}$$

The system of equations (6) for the dimensionless stream and temperature functions Φ and Θ were integrated numerically on an ES-1033 computer. The derivatives in the equations were replaced by their finite-difference relations. For the equation of motion the boundary problem reduced to a Cauchy problem which was solved by a fourth-order Runge–Kutta method. The heat-transfer equation was solved by a marching method. The result was dimensionless profiles of velocity and temperature, the function A, and the specific heat fluxes for various values of the parameters f_1, γ_1, $\bar{\gamma}_1$. Figure 1 shows some results of the numerical integration of the universal equations (6). In particular, it shows curves of the universal functions A as a function of the values f_1 and the specific heat flux to the heated surface $\dot{\Theta}(0)$ for various values of f_1, γ_1, $\bar{\gamma}_1$. For $-20 < f_1 < 4$ one can approximate to $A(f_1)$ satisfactorily by linear function. Then the equation for determining the unknown film thickness acquires the form, convenient for integration

$$\frac{gh^3}{\nu Q} = 3 - \frac{54}{35}\,\frac{Qh'}{\nu}. \tag{7}$$

From the solutions of the universal equations (6) obtained, one can evaluate all the characteristics of the heat-transfer process: the heat flux at the outer edge of the film, the heat flux to the heated surface, and the average temperatures over sections. We now examine one of the basic characteristics of the process, the heat-transfer coefficient to a flowing liquid film, defined to be

$$\alpha = -\left.\frac{\partial T}{\partial y}\right|_{y=0} \Big/ (T_W - T_S). \tag{8}$$

The results of the calculations showed that the influence of the parameter f_1 on the heat transfer is negligible, and appears only as a change of the thickness of the film. Thus, the heat-transfer process in the liquid film is less sensitive to deformation of the velocity profile due to inertia forces than are the hydrodynamic characteristics of the flow. The heat transfer in liquid film flow is mainly influenced by the nature of the temperature variation of the temperature of the heated surface and the free film surface. For values of the parameters $|\gamma_1| < 1.5$, $|\bar{\gamma}_1| < 1.5$ the results of the heat-transfer coefficient calculations are approximated satisfactorily by the formula, convenient for practical calculations

$$\alpha/\alpha_0 = 1 + 0.2\gamma_1 + 0.175\bar{\gamma}_1, \tag{9}$$

where $\alpha_0 = \lambda/h$ is the heat transfer coefficient for hydrodynamically stabilized flow of a film constant temperatures T_W and T_S. In each specific case the parameters γ_1 and $\bar{\gamma}_1$ are determined from the temperatures at the outer edge of the film and on the heated surface from the heat-transfer conditions.

Putting T_S = const ($\bar{\gamma}_1 = 0$), we now analyze the influence of the heated surface not being isothermal on the heat transfer to the liquid film. For example, positive temperature gradients of the heated surface ($\gamma_1 > 0$) lead to an increase in the heat-transfer intensity compared with isothermal flow, while negative gradients ($\gamma_1 < 0$) lead to reduced intensity. For a specific value of the parameter γ_1 deformations of the temperature profile lead to a change of the direction of the specific flux, analogously to heat transfer in a boundary layer [4]. The greatest increase of heat-transfer intensity will be observed for positive gradients at the outer boundary of the film ($\bar{\gamma}_1 > 0$), i.e., at a maximum value of the temperature head.

NOTATION

h, film thickness; L, characteristic linear dimension; x, y, Cartesian coordinates; T, temperature; g, acceleration due to gravity; ν, coefficient of kinematic viscosity; a, thermal diffusivity; λ, thermal conductivity; Q_0, specific volume flow rate; Pr, Prandtl number.

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